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ORIGINAL ARTICLE

Travel time model for shuttle-based storage and retrieval systems

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Abstract This paper presents analytical travel time model for the computation of travel (cycle) time for shuttle-based storage and retrieval systems (in continuation SBS/RS). The proposed model considers the operating characteristics of the elevators lifting table and the shuttle carrier, such as acceleration and deceleration and the maximum velocity. Assuming uniform distributed storage rack locations and using the probability theory, the expressions of the cumulative distribution functions with which the mean travel time is calculated, have been determined. The proposed model enables the calculation of the mean travel (cycle) time for the single and dual command cycles, from which the performance of SBS/RS can be evaluated. The approximation model and a simulation model of SBS/RS have been used to compare the performances of the proposed analytical travel time model. The analysis shows that regarding all examined types of SBS/RS, the results of proposed analytical travel time model for SBS/RS correlate with the results of simulation models of SBS/RS.

Keywords Automated warehouses · Shuttle-based storage and retrieval systems · Tier-captive system · Analytical modelling · Simulation

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Abbreviations

AVS/RS	Autonomous vehicle storage and retrieval systems
AS/RS	Automated storage and retrieval systems
CO_2	Carbon dioxide
DCC	Double command cycle
FEM	European Federation of Materials Handling
I/O	Input and output location
ES	One-way travel time component
RC	Rack configuration of the SBS/RS
SOQN	Semi-open queuing network
SBS/RS	Shuttle-based storage and retrieval systems
SCC	Single command cycle
SR	Storage rack
TB	Travel-between time component
TUL	Transport unit load

1 Introduction

Technological developments in warehouses have changed processes of storage operations, which reflect in short response times of the storage or retrieval of goods, the reduction of stocks and the volume of storage work as well as the automation of the entire warehouse management. Numerous companies are replacing costly and lasted traditional warehouses with automated ones.

An important part of automated warehouses is presented by a relatively new technology called shuttle-based storage and retrieval systems (SBS/RS), which are widely used in many fields of industry, where the basic transport unit load (TUL) is presented by a tote (plastic container).

The SBS/RS is composed of multiple parallel aisles of storage racks (SR), elevator (lift) intended for each aisle of the SR, tier-captive shuttle carriers, input and output (I/O) location, buffer position in each tier and roller conveyors. Advantages of the application of SBS/RS are efficient utilisation of the warehouse space, reduction of damage and loss of goods, increased control upon storage and retrieval of goods and decrease in the number of warehouse workers. On the other hand, SBS/RS require a high initial investment compared to other types of automated warehouses. Therefore, a careful design of SBS/RS is crucial for the SBS/RS to be successful. The performance of the SBS/RS is often evaluated by the number of totes per hour, which may be stored, and the number of totes, which may be retrieved—the throughput capacity of the system. Due to the increasing requests for higher throughput capacities and shorter response times in handling the orders, special designs of lift (elevator), which can carry two lifting tables simultaneously, have been constructed. Many warehousing equipment producers have begun to offer such lift that can receive up to two totes simultaneously (application of two lifting tables) and consequently higher throughput capacities of the SBS/RS can be achieved. SBS/RS have been the subject of not so many researchers over the past few years, regardless the fact that their intensive development has begun approximately 10 years ago. Based to our knowledge, there are only a few scientific papers that directly relates to SBS/RS. More has been done for the research field of autonomous vehicles storage and retrieval systems (AVS/RS).

AVS/RS is first studied by Malmborg [20]. Malmborg [21] proposes a state equation model for predicting the proportion of dual command cycles (DCC) in AVS/RS. Although there are some limitations in the proposed model, it provides a useful tool for estimating DCC in AVS/RS.

Fukunari and Malmborg [10] develop an efficient cycle time model for AVS/RS and compare their performance with crane-based AS/RS. Their model is based on an iterative computational scheme considering random storage assumptions and queuing model approximations. This model also improves upon earlier models by scaling efficiently for large problems. The procedure is shown using realistically sized problems. Later, Fukunari and Malmborg [11] proposed a queuing network approach to predict resource utilisations in AVS/RS. Although this technique lacks the capability for modelling the transaction queuing process directly, it provides reasonably accurate estimates of the resource utilisation measures that are essential to the process of system conceptualisation.

Kuo et al. [12] developed a computationally efficient cycle time model for AVS/RS estimating resource utilisation. They solved 12 different scenarios to show the performance of the model. Although the model shows some substantial errors, the model can provide an accuracy level for estimates of vehicle utilisation and system cost.

Later, Kuo et al. [13] studied a queuing network to estimate performance measures for AVS/RS where class-based storage policies may be used to mitigate the cycle-time inflation effects of storage. The model is capable of efficiently generating estimates of AVS/RS resource utilisation based on alternative class-based storage schemes.

Zhang et al. [26] studied variance-based approximation for waiting times in AVS/RS. They modelled the system using a series of queuing approximations, by dynamically selecting between three alternative queuing approximations based on the squared coefficient of variation of transaction inter-arrival times. The proposed model's results show greater accuracy in estimation of transaction waiting times, thereby enabling more effective design conceptualisation. They also incorporate the proposed models in online tools for helping warehouse designers and analysts develop alternate AS/RS and AVS/RS warehouse configurations.

Recently, Ekren et al. [5], Ekren [3] and Ekren and Heragu [9] have studied simulation-based performance evaluation of AVS/RS. They study near optimum rack configuration design under pre-defined scenarios of number of vehicles and lifts in the system using simulation-based regression analysis [4] and implement a design of experiments for an AVS/RS to identify factors affecting its performance [5].

Roy et al. [24] proposed a semi-open queuing network (SOQN) model to evaluate design trade-offs in a single tier of an AVS/RS. Their model captures the effect of location of the vehicles within a tier using multiple vehicle classes and class switching probabilities. Because exact solutions to the semi-open queuing network are not available, they proposed a decomposition approach. Model results suggest benefits of having multiple zones due to reduction in travel time along the cross-aisle.

Ekren et al. [6, 7] also study SOQN to model an AVS/RS. They use their pre-proposed extended algorithm [8] to calculate the performance measure of the system. By this study, they showed that the system could be modelled by SOQN, efficiently.

The most related paper to the studied system is completed by Carlo and Vis [2]. They study a type of SBS/RS where there are two non-passing lifting systems mounted along the rack. They focus on scheduling problem where two (piecewise linear) functions are introduced to evaluate candidate solutions.

Marchet et al. [22] study main design trade-offs for AVS/RS using simulation. They complete their study for several warehouse design scenarios for two types of AVS/RS configurations: tier captive and tier-to-tier vehicles.

Lerher at al. [17, 18] have developed analytical travel time models for multi-aisle AS/RS considering the operating characteristics of the S/R machine. With proposed analytical travel time models, average cycle time, from which the performance of multi-aisle AS/RS can be evaluated, was determined.

Lerher et al. [14, 15] study multi-objective optimisation for automated warehouses. For the optimisation of decision variables in objective functions, the method with genetic algorithms has been used.

Recently, Lerher [19] and Lerher et al. [16] have studied energy regeneration and energy efficiency models for SBS/RS. The proposed models enable reduction of energy consumption and consequently the CO_2 emission, which is vital from economic and environmental point of view. It is sincerely believed that the energy and environment aspect will indubitably bring changes into planning of warehouses and will mean great challenge for those, who are engaged in the planning process.

Sari at al. [25] have studied experimental validation of travel time models for shuttle-based automated storage and retrieval system.

Different from the existing studies, we extend the previous SBS/RS models [14–16, 19] by implementing the analytical (continuous) travel time model for SBS/RS (Lerher at al. [17]). The proposed model considers the real operating characteristics of the elevators lifting table and the shuttle carrier under the condition of the tier-captive SBS/RS. Because the layout of the SBS/RS has a major influence on the performance and the efficiency of the SBS/RS, nine different rack configurations of SBS/RS with five velocity profiles are presented. The proposed analytical (continuous) travel time model for SBS/RS has also been validated with discrete event simulations.

This paper is organised as follows: In Section 2, the description of SBS/RS under study is given. In Section 3, the proposed analytical travel time model for SBS/RS to calculate the mean cycle time for the single and dual command cycle, considering the operating characteristics of the elevators lifting table and the shuttle carrier, are presented. The approximation model for SBS/RS is presented in Section 4. The simulation model of SBS/RS is presented in Section 5. In Section 6, the performance of the proposed model according to the approximation model and simulation results is evaluated. Finally, the conclusion is given in Section 7.

2 Shuttle-based storage and retrieval system

A typical installation of SBS/RS consists of a warehouse for totes, with an elevator in each aisle and the shuttle carriers operating in each single tier (see Fig. 1). Installation heights of 20 m or more can be achieved, and typical operating (pick) aisle for standard totes can be about 0.5 m wide. The warehouse management system monitors the status of all components in the system (elevators lifting tables, shuttle carriers, buffer positions, etc.), and, based on the warehouse inventory and movement requirements, it plans the work to be carried out. The elevator consists of a vertical mast or a pair of masts supporting the lifting table, which can be raised or lowered. Newer designs of SBS/RS have installations of two lifting tables, which can work independently of each other. Thus, higher throughput capacity can be achieved. The elevator is feeding the buffer positions, which are set at the beginning of each tier. In each tier, there is a shuttle carrier that can store and retrieve totes (tier-captive system) to/from the tier. Since in each tier there is a single shuttle carrier, the elevator is most often the bottleneck of the SBS/RS. The amount of required storage locations depends on the designed inventory holding capacity.

The initial design of SBS/RS is determined by the total amount of totes movement in a given period of time (throughput capacity of the system). Many producers of the warehouse equipment, such as Schäfer, Knapp, Siemens Dematic, Stöcklin, and many others, have begun to offer SBS/RS for nearly 10 years. The main benefits using SBS/RS are the following: subsequent expansion is possible at any time, optimum use is made of space due to minimal overrun dimensions and high throughput is achieved compared to mini-load AS/RS.

When developing proposed analytical travel time model for SBS/RS, the following assumptions were considered:

- The SBS/RS is divided into two sides in a picking aisle. Therefore, totes can be stored in either side in a tier (see Fig. 2).
- The input/output (I/O) location of the SBS/RS is located at the first tier, next to the lift location (see Fig. 2).
- The storage rack SR is divided by columns and tiers. At each tier, there are two buffer positions (left and right) and a single shuttle carrier (the application of the tier-captive system) (see Fig. 2).
- The elevator manipulates two lifting tables independently, one of which is located at the left side and the other one is located at the right side of the elevator. Each lifting table can serve one tote at a time (see Fig. 2).
- The elevator and shuttle carrier operate on the basis of single command cycles (SCC) and dual command cycles (DCC).
- Drive characteristics (v_y, a_y) of the elevators lifting tables as well as the height H of the storage rack are known priori.
- Drive characteristics of the shuttle carrier (v_x, a_x) as well as the length L of the storage racks are known priori.
- The height H and length L of the storage racks are large enough for the elevators lifting table and the shuttle carrier to reach their maximum velocity v_{max} in the vertical and in the horizontal direction.

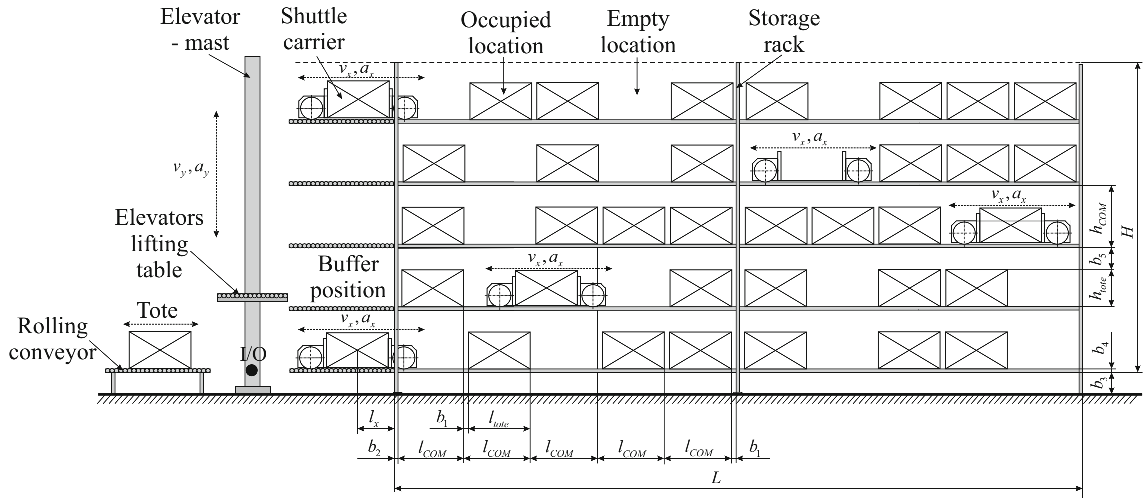

Fig. 1 Tier-captive SBS/RS

- Randomised assignment policy is considered, which means that any storage position is equally likely to be selected for storage or retrieval location to be processed.

In addition, the following notation and symbols are introduced throughout the paper:

Operational parameters:

a	Acceleration/deceleration
a_y	Acceleration/deceleration of the elevators lifting table
a_x	Acceleration/deceleration of the shuttle carrier
a_x^+	Acceleration of the shuttle carrier
a_y^+	Acceleration of the elevators lifting table
T	Arrival time at a destination
r_i	Closed location
$y_{(k)}$	Coordinate of location k in the vertical direction.
$x_{(k)}$	Coordinate of location k in the horizontal direction
$F(t)$	Cumulative distribution function
a_x^-	Deceleration of the shuttle carrier
a_y^-	Deceleration of the elevators lifting table
b_3	Deviation of the first storage compartment from the floor
$s(t)$	Displacement (distance travelled) as a function of travel time
$d(T)$	Distance moved during time T
$E(\mathrm{ES})_{\mathrm{SCAR}}$	Expected one way travel time of the shuttle carrier.
$E(\mathrm{SCC})_{\mathrm{SCAR}}$	Expected single command cycle time of the shuttle carrier.
$E(\mathrm{TB})_{\mathrm{SCAR}}$	Expected travel between time of the shuttle carrier.
$E(\mathrm{DCC})_{\mathrm{SCAR}}$	Expected dual command cycle time of the shuttle carrier.
$E(\mathrm{ES})_{\mathrm{LIFT}}$	Expected one way travel time of the elevators lifting table.
$E(\mathrm{SCC})_{\mathrm{LIFT}}$	Expected single command cycle time of the elevators lifting table.
$E(\mathrm{TB})_{\mathrm{LIFT}}$	Expected travel between time of the elevators lifting table.

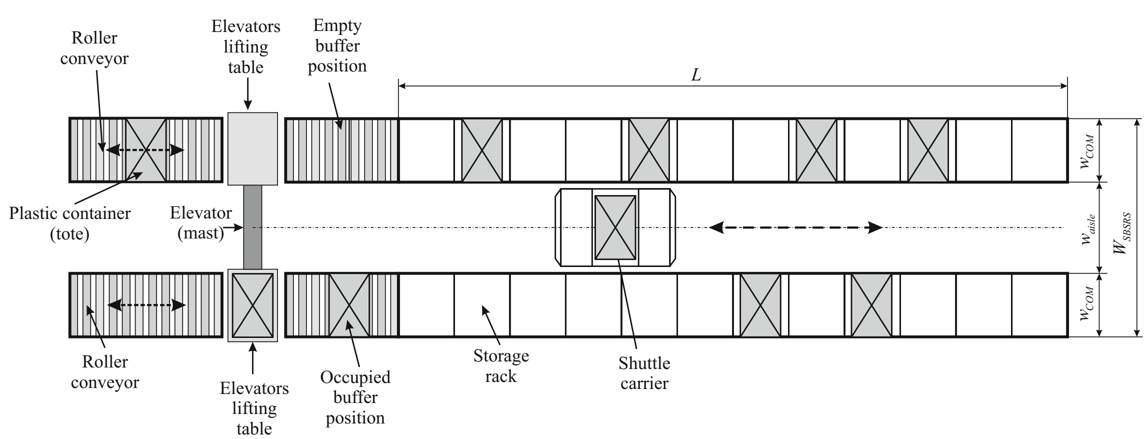

Fig. 2 Layout of the tier-captive SBS/RS with two lifting tables

$E(\mathrm{DCC})_{\mathrm{LIFT}}$	Expected dual command cycle time of the elevators lifting table.
α	Fillgrade factor
h_{tote}	Height of the tote
h_{COM}	Height of the tier
b_4	Height of rack beams
h_{tier}	Height of the tier
H	Height of the storage rack
T_{kx}	Horizontal travel time depending of the regions for the travel type I and II
t_{Bx}	Horizontal interleave time required to travel between the storage and the retrieval locations
l_{tote}	Length of the tote
l_{COM}	Length (depth) of the column
l_x	Length from the $\mathrm{I/O}_{\mathrm{tier}(i)}$ to the first upright frame
L	Length of the storage rack
$T(\mathrm{SCC})_{\mathrm{SCAR}}$	Mean single command cycle time of the shuttle carrier
$T(\mathrm{DCC})_{\mathrm{SCAR}}$	Mean dual command cycle time of the shuttle carrier
$T(\mathrm{SCC})_{\mathrm{LIFT}}$	Mean single command cycle time of the elevators lifting table
$T(\mathrm{DCC})_{\mathrm{LIFT}}$	Mean dual command cycle time of the elevators lifting table
v_y	Maximum velocity of the elevator's lifting table in the vertical direction
v_x	Maximum velocity of the shuttle carrier in the horizontal direction
$v_{\max}$	Maximum velocity
l	Minimum distance necessary to reach v_x
h	Minimum distance necessary to reach v_y
A	Number of aisles
C	Number of columns
M	Number of tiers
s_i	Open location
$v(t_p)$	Peak velocity at time t_p
λ	Performance comparison between analytical models and the discrete model
$t_{\mathrm{P/S}}$	Pick-up and set-down times of the elevators lifting table/shuttle carrier
Pr	Probability
$h(r_i)$	Probability density function
r_i	Range
t_x	Required travel time for the shuttle carrier to reach l
T_x	Required travel time for the shuttle carrier to reach L
t_y	Required travel time for the elevator's lifting table to reach h
T_y	Required travel time for the elevator's lifting table to reach H
b_1	Safety addition to the width of the storage compartment
b_5	Safety addition to the height of the storage compartment
S_i	Set of totes to be stored
R_i	Set of totes to be retrieved
b_2	Thickness of the upright frame
t_p	Time necessary to reach the peak velocity
$t_{\mathrm{I/O,tier}(i)}$	Travel time of the elevators lifting table from the I/O location to the ith tier
$t_{\mathrm{tier}(i),\mathrm{tier}(j)}$	Travel time of the elevators lifting table from the ith tier to the jth tier
$t_{\mathrm{tier}(j),\ \mathrm{I/O}}$	Travel time of the elevators lifting table from the jth tier to the I/O location
$t_{\mathrm{I/O}_{\mathrm{tier}(i)},s_1}$	Travel time of the shuttle carrier from the $\mathrm{I/O}_{\mathrm{tier}(i)}$ to the storage location s_1
$t_{s_1,\ r_1}$	Travel time of the shuttle carrier from the storage location s_1 to the retrieval location r_1
$t_{r_1,\mathrm{I/O}_{\mathrm{tier}(i)}}$	Travel time of the shuttle carrier from the retrieval location r_1 to the $\mathrm{I/O}_{\mathrm{tier}(i)}$
τ_y	Travel time function of the elevators lifting table
τ_x	Travel time function of the shuttle carrier
t	Travel time
z	Variable
t_{kx}	Variable share of the horizontal travel time
t_{ky}	Variable share of the vertical travel time
vp_i	Velocity profile
$v(t)$	Velocity at time t
T_{ky}	Vertical travel time depending of the regions for the travel type I and II
t_{By}	Vertical interleave time required to travel between the storage and the retrieval locations
Q	Warehouse volume
w_{tote}	Width of the tote
w_{COM}	Width of the column
w_{aisle}	Width of the aisle
$W_{\mathrm{SBS/RS}}$	Width of the SBS/RS

3 Analytical travel time model for SBS/RS

In continuation, analytical travel time model for SBS/RS will be in detail presented. Since the proposed model is based on the velocity profile of the elevators lifting table and the shuttle carrier, the fundamentals of travel time will be first presented. Later on, the expressions for the single and dual command cycle for the elevators lifting table and the shuttle carrier will be presented.

3.1 The fundamentals of travel time

Two types of velocity profiles can be distinguished depending on whether the obtained peak velocity $v(t_p)$ is less than $v_{\max}$

(type I) or equal to v_{max} (type II) (see Fig. 3). It can be verified that time $T<2v_{max}/a$ for type I and $T>2v_{max}/a$ for type II [18].

3.1.1 Travelling for type I ($T<2v_{max}/a$)

The velocity in dependence of time $v(t)$ equals the following expression [18]:

$$v(t) = \begin{cases} at, & t\in(0,t_p) \\ -a(t-T), & t\in(t_p,T) \end{cases} \tag{1}$$

The distance in dependence of time $d(T)$ equals the following expression [18]:

$$d(T) = \int_0^T v(t)\mathrm{d}t = \frac{aT^2}{4} \tag{2}$$

Because of the acceleration and deceleration are equal in magnitude, the time necessary to reach the peak velocity equals $t_p=T/2$. For the verification of expression 2, see Appendix 1.

3.1.2 Travelling for type II ($T>2v_{max}/a$)

The velocity in dependence of time $v(t)$ equals the following expression [18]:

$$v(t) = \begin{cases} at, & t\in(0,t_p) \\ v_{\max}, & t\in(t_p,T-t_p) \\ -a(t-T), & t\in(T-t_p,T) \end{cases} \tag{3}$$

The distance in dependence of time $d(T)$ equals the following expression [18]:

$$\mathrm{d}(T) = \int_0^T v(t)\mathrm{d}t = v_{\max}T - \frac{{v_{\max}}^2}{a} \tag{4}$$

For the verification of expression 4, see Appendix 1.

3.2 Single command cycle in SBS/RS

The operation of the SCC encompasses either the storage or the retrieval sequence (see Fig. 4). With regard to the SBS/RS, the SCC in the SBS/RS combines (i) lifting of the elevators lifting table to the ith tier in the vertical direction and (ii) travelling of the shuttle carrier in the ith tier in the horizontal direction. The efficiency of the SCC in the SBS/RS is therefore based on:

- Lifting of the elevators lifting table in the vertical direction
- Travelling of the shuttle carrier in the horizontal direction.

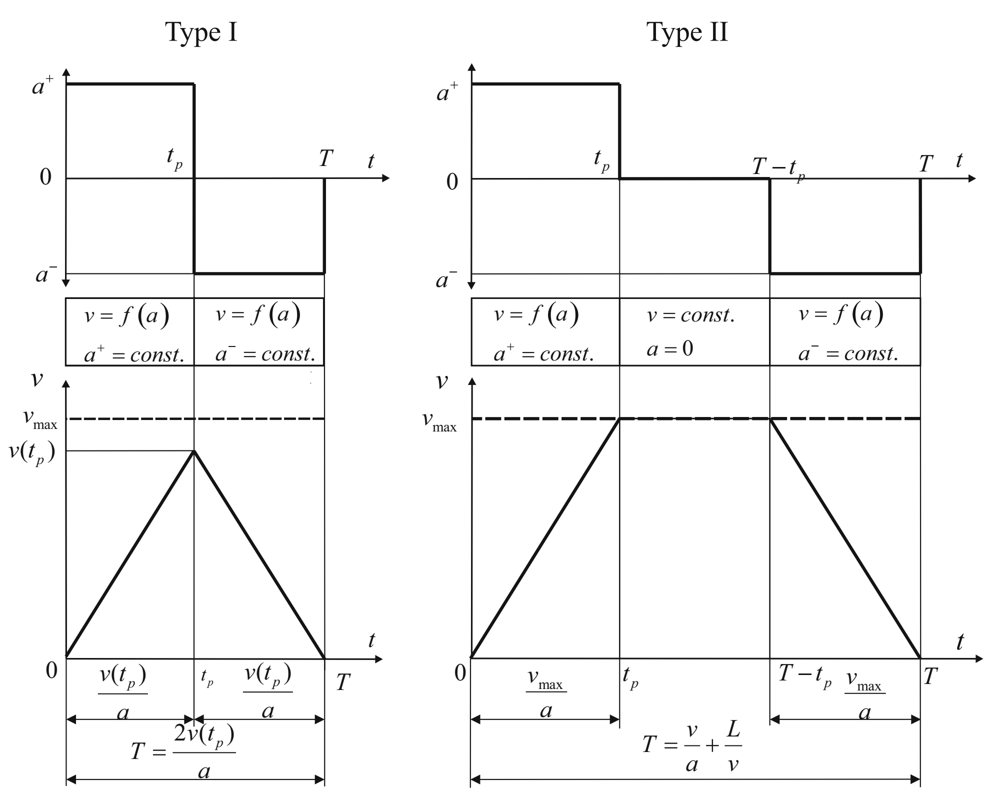

Fig. 3 Velocity–time relationship [18]

3.2.1 Lifting of the elevators lifting table in the vertical direction

In the case of SCC, the elevator lifts the lifting table with a tote to the ith tier, unloads the tote and moves back to the I/O location, which is positioned at the first tier. The same sequence can also be performed in reverse order [19]. One-way travel time $(\mathrm{ES})_{\mathrm{LIFT}}$ corresponds to the variable travel time t for lifting of the elevators lifting table with a tote from I/O location to the selected ith tier (see Fig. 4). As regards the condition of uniform distribution of tiers in the SBS/RS, the cumulative distribution function $F_y(t)$ is accomplished.

The cumulative distribution function $F_y(t)$ is distinguished according to the following condition:

(a) Lifting of the elevators lifting table with a tote for type I, where $t_y=2v_y/a_y$ is the required travel time for the elevators lifting table to reach $h=v_y^2/a_y$.
(b) Lifting of the elevators lifting table for type II, where $T_y=H/v_y+v_y/a_y$ is the required travel time for the elevators lifting table to reach H.

- Cumulative distribution function $F_y(t)$ for lifting of the elevators lifting table in the SBS/RS

$$F_y(t) = \begin{cases} \dfrac{t^2 a_y}{4H}, & \left(0 \le t \le \dfrac{2v_y}{a_y}\right) \\ \dfrac{v_y t}{H} - \dfrac{v_y^2}{a_y H}, & \left(\dfrac{2v_y}{a_y} \le t \le \dfrac{H}{v_y} + \dfrac{v_y}{a_y}\right) \end{cases} \qquad (5)$$

For the verification of expression 5, see Appendix 2.

- Cumulative distribution function $F(t)$

The cumulative distribution function $F(t)$ depends on the relationships among the values of the following parameters: v_y, a_y and H. Therefore, $F(t)$ can be specified under the following condition:

$$F(t) = F_y(t), \qquad \left(0 \le t \le T_y\right) \qquad (6)$$

The expected one way travel time $E(\mathrm{ES})_{\mathrm{LIFT}}$ for lifting of the elevators lifting table in the SBS/RS is equal to the next expression:

$$E(\mathrm{ES})_{\mathrm{LIFT}} = \int_0^{T_y} [1-F(t)]\mathrm{d}t \qquad (7)$$

- Expected single command cycle time for the elevators lifting table

According to lifting of the elevators lifting table in the vertical direction, the expected single command cycle time for the SBS/RS is represented with the following expression:

$$E(\mathrm{SCC})_{\mathrm{LIFT}} = 2t_{\mathrm{P/S}} + 2E(\mathrm{ES})_{\mathrm{LIFT}} \qquad (8)$$

where $t_{\mathrm{P/S}}$ stands for the pick up and set down times for the elevators lifting table.

Pick-up and set-down times $t_{\mathrm{P/S}}$ of the elevators lifting table in case of SCC relates to pick up the tote from the I/O location at the first tier and set down the tote at the buffer position in the ith tier.

3.2.2 Travelling of the shuttle carrier in the horizontal direction

Under travelling of the shuttle carrier in the ith tier, the shuttle carrier is capable of visiting a single storage or retrieval location (see Fig. 5). The travel time depends on the kinematics properties of the shuttle carrier, the length L of the tier

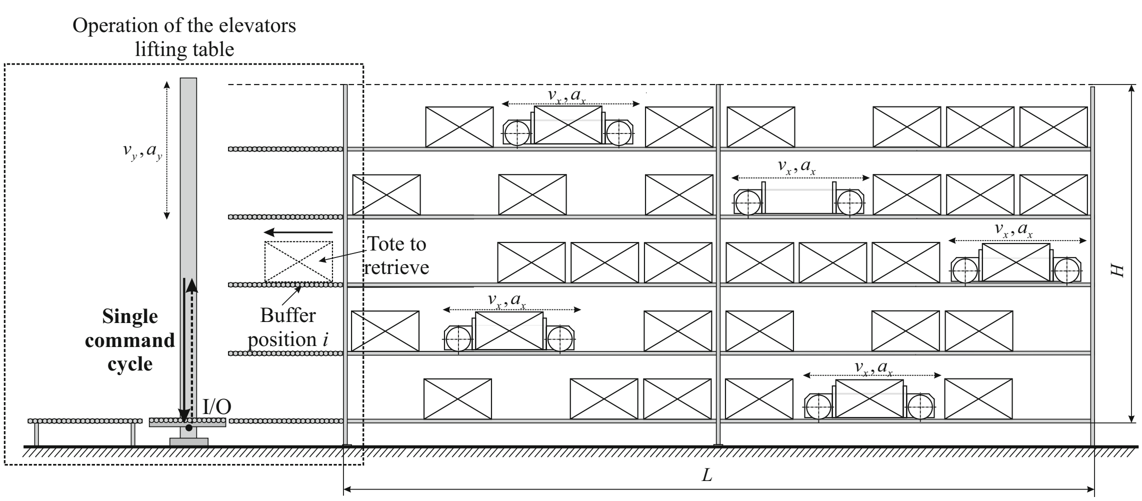

Fig. 4 Single command cycle of the elevators lifting table in the SBS/RS

(storage rack) and the selected storage assignment policy. One-way travel time $(\mathrm{ES})_{\mathrm{SCAR}}$ corresponds to the variable travel time t for travelling from the $\mathrm{I/O}_{\mathrm{tier}(i)}$ location to any randomly selected location in the ith tier (see Fig. 5). As regards the condition of uniform distribution of storage locations in the SBS/RS, the cumulative distribution function $F_x(t)$ is accomplished. The cumulative distribution function $F_x(t)$ is distinguished according to the following condition:

(i) Shuttle carrier travelling for type I, where $t_x=2v_x/a_x$ is the required travel time for the shuttle carrier to reach $l=v_x^2/a_x$.
(ii) Shuttle carrier travelling for type II, where $T_x=L/v_x+v_x/a_x$ is the required travel time for the shuttle carrier to reach L.

- Cumulative distribution function $F_x(t)$ for travelling of the shuttle carrier in the horizontal direction

$$F_x(t)=\begin{cases}\dfrac{a_x t^2}{4L}, & \left(0\le t\le \dfrac{2v_x}{a_x}\right)\\[2ex] \dfrac{v_x t}{L}-\dfrac{v_x^2}{a_x L}, & \left(\dfrac{2v_x}{a_x}\le t\le \dfrac{L}{v_x}+\dfrac{v_x}{a_x}\right)\end{cases} \tag{9}$$

For the verification of expression 9, see Appendix 2.

- Cumulative distribution function $F(t)$

The cumulative distribution function $F(t)$ is defined according to the travelling of the shuttle carrier in the horizontal direction and depends on the relationships among the values of the following parameters: v_x, a_x and L. Therefore $F(t)$ can be specified with:

$$F(t)=F_x(t), \qquad (0\le t\le T_x) \tag{10}$$

The expected one-way travel time under single command cycle $E(\mathrm{ES})_{\mathrm{SCAR}}$ is equal to the next expression:

$$E(\mathrm{ES})_{\mathrm{SCAR}}=\int_0^{T_x}[1-F(t)]\mathrm{d}t \tag{11}$$

- Expected single command cycle time for the shuttle carrier

According to travelling of the shuttle carrier in the horizontal direction, the expected single command cycle time for the SBS/RS is represented with the following expression:

$$E(\mathrm{SCC})_{\mathrm{SCAR}}=2t_{\mathrm{P/S}}+2E(\mathrm{ES})_{\mathrm{SCAR}} \tag{12}$$

where $t_{\mathrm{P/S}}$ stands for the pick-up and set-down times for the shuttle carrier.

Pick-up and set-down times $t_{\mathrm{P/S}}$ of the shuttle carrier in case of SCC relates to pick up the tote from the $\mathrm{I/O}_{\mathrm{tier}(i)}$ buffer position and set down the tote in the storage location in the ith tier of the SR.

3.3 Dual command cycle in SBS/RS

The operation of the DCC encompasses the storage and the retrieval sequence. With regard to the SBS/RS, the DCC in the SBS/RS combines (i) lifting of the elevators lifting table in the vertical direction and (ii) travelling of the shuttle carrier in the horizontal direction.

The efficiency of the DCC in the SBS/RS is therefore based on:

– Lifting of the elevators lifting table in the vertical direction
– Travelling of the shuttle carrier in the horizontal direction.

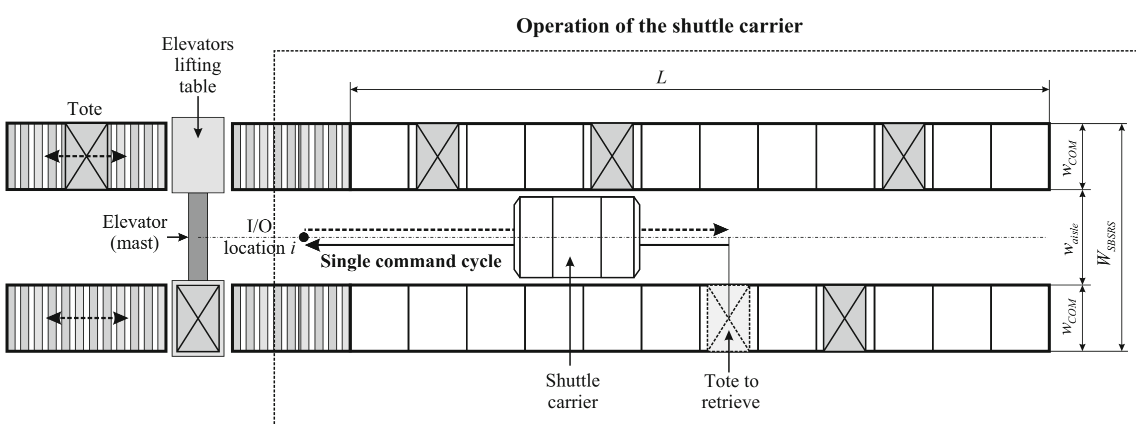

Fig. 5 Single command cycle of the shuttle carrier operating in the ith tier of the SBS/RS

3.3.1 Lifting of the elevators lifting table in the vertical direction

In the case of DCC, the elevator moves the lifting table with the tote to the ith tier, unloads the tote and moves further to the jth tier, where the tote is retrieved (see Fig. 6). After loading the tote at the jth tier, the elevator moves the lifting table back to the I/O location [19].

Travel time for DCC corresponds to the travel time for SCC to the randomly selected ith tier and travel-between $(\mathrm{TB})_{\mathrm{LIFT}}$ time component for DCC, where the retrieval request occurs in the ith tier or in the jth tier (Fig. 6).

By definition DCC involves two randomly selected tiers i and j, one representing the storage point at the ith tier and the other representing the retrieval point at jth tier. The expected travel-between $(\mathrm{TB})_{\mathrm{LIFT}}$ time for DCC for two randomly selected tiers i and j equals $E(\mathrm{TB})_{\mathrm{LIFT}}$. According to the condition of uniform distribution of all tiers, the cumulative distribution function $F_y(t)$ is accomplished.

The cumulative distribution function is distinguished according to the following condition:

(i) Lifting of the elevators lifting table for type I, where $t_y=2v_y/a_y$ is required travel time for the elevators lifting table to reach $h=v_y^2/a_y$

(ii) Lifting of the elevators lifting table for type II, where $T_y=H/v_y+v_y/a_y$ is required travel times for the elevators lifting table to reach H.

- Cumulative distribution function $F_y(t)$ for lifting of the elevators lifting table in the vertical direction

$$F_y(t)=\begin{cases} \frac{a_y}{2H}t^2-\frac{a_y^2}{16H^2}t^4, & \left(0\le t\le\frac{2v_y}{a_y}\right) \\ -\frac{v_y^2}{H^2}t^2+\left[\frac{2v_y^3}{a_yH^2}+\frac{2v_y}{H}\right]t-\frac{2v_y^2}{a_yH}-\frac{v_y^4}{a_y^2H^2}, & \left(\frac{2v_y}{a_y}\le t\le\frac{H}{v_y}+\frac{v_y}{a_y}\right) \end{cases} \tag{13}$$

For the verification of expression 13, see Appendix 3.

- Cumulative distribution function $F(t)$

The cumulative distribution function $F(t)$ is defined according to lifting of the elevators lifting table in the vertical direction and depends on the relationships among the values of the following parameters: v_y, a_y and H. Therefore, $F(t)$ can be specified with:

$$F(t)=F_y(t),\qquad \left(0\le t\le T_y\right) \tag{14}$$

The expected travel-between $E\,(\mathrm{TB})_{\mathrm{LIFT}}$ time for DCC for two randomly selected tiers i and j is equal to the following expression:

$$E(\mathrm{TB})_{\mathrm{LIFT}}=\int_0^{T_y}[1-F(t)]\mathrm{d}t \tag{15}$$

- Expected dual command cycle time for the elevators lifting table

According to lifting of the elevators lifting table in the vertical direction, the expected dual command cycle time for the SBS/RS is represented with the following expression:

$$E(\mathrm{DCC})_{\mathrm{LIFT}}=4t_{\mathrm{P/S}}+E(\mathrm{SCC})_{\mathrm{LIFT}}+E(\mathrm{TB})_{\mathrm{LIFT}} \tag{16}$$

where $t_{\mathrm{P/S}}$ stands for the pick up and set down times for the lift.

Pick-up and set-down times $t_{\mathrm{P/S}}$ of the elevators lifting table in case of DCC relates to pick up the tote from the I/O location, set down the tote at the storage location in the buffer position at the ith tier, pick up the tote from the retrieval location in the buffer position at the jth tier and finally set down the tote at the I/O location.

3.3.2 Travelling of the shuttle carrier in the horizontal direction

In case of the shuttle carrier, the operation of DCC considers storage and retrieval processes at a time (see Fig. 7). Recall that in DCC, the shuttle carrier travels to two storage locations between successive returns to the $\mathrm{I/O}_{\mathrm{tier}(i)}$. After completing a given storage request at location s_1, the shuttle carrier moves directly to another location for the next retrieval request at location r_1 without returning to the $\mathrm{I/O}_{\mathrm{tier}(i)}$.

Therefore, the travel time for DCC corresponds to the travel time for SCC in the randomly selected ith tier and travel-between $(\mathrm{TB})_{\mathrm{SCAR}}$ time for DCC, where the retrieval request occurs in the same ith tier (application of the tier-captive shuttle system).

By definition DCC involves two randomly selected locations in the ith tier, one representing the storage location s_1 and the other representing the retrieval location r_1. The expected travel-between $(\mathrm{TB})_{\mathrm{SCAR}}$ time for DCC for two randomly selected locations s_1 and r_1 is the same as $E(\mathrm{TB})_{\mathrm{SCAR}}$. According to the condition of uniform distribution of storage locations in the SR, the cumulative distribution functions $F_x(t)$ is accomplished.

The cumulative distribution function is distinguished according to the following condition:

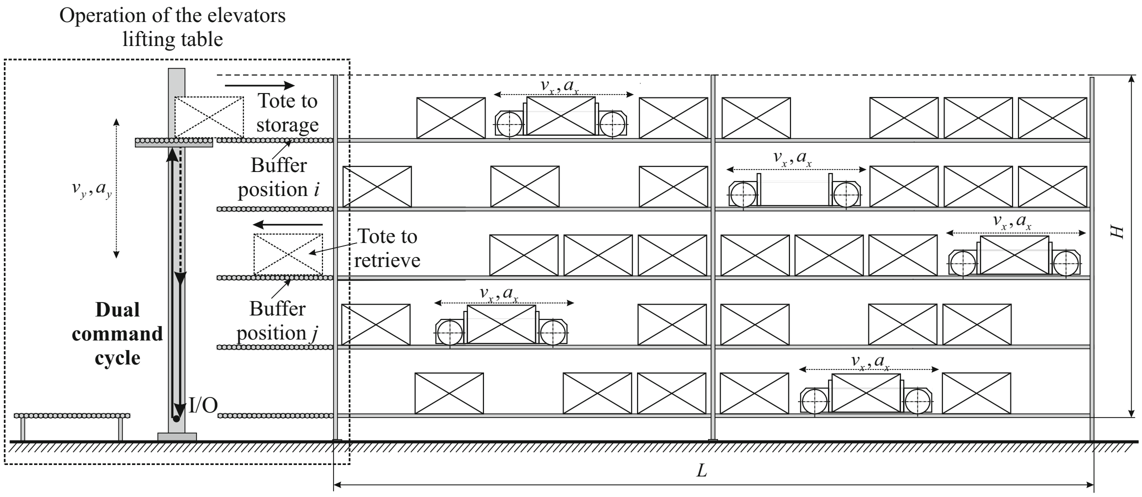

Fig. 6 Dual command cycle of the elevators lifting table in the SBS/RS

(i) Shuttle carrier travelling for type I, where $t_x=2v_x/a_x$ is required travel time for the shuttle carrier to reach $l=v_x^2/a_x$.
(ii) Shuttle carrier travelling for type II, where $T_x=L/v_x+v_x/a_x$ is required travel time for the shuttle carrier to reach L.

- Cumulative distribution function $F_x(t)$ for travelling of the shuttle carrier in the horizontal direction

$$F_x(t)=\begin{cases}\frac{a_x}{2L}t^2-\frac{a_x^2}{16L^2}t^4, & \left(0\le t\le\frac{2v_x}{a_x}\right)\\ -\frac{v_x^2}{L^2}t^2+\left(\frac{2v_x^3}{a_xL^2}+\frac{2v_x}{L}\right)t-\frac{2v_x^2}{a_xL}-\frac{v_x^4}{a_x^2L^2}, & \left(\frac{2v_x}{a_x}\le t\le\frac{L}{v_x}+\frac{v_x}{a_x}\right)\end{cases} \tag{17}$$

For the verification of expression 17, see Appendix 3.

- Cumulative distribution function $F(t)$

The cumulative distribution function $F(t)$ is defined according to the travelling of the shuttle carrier in the horizontal direction and depends on the relationships among the values of the following parameters: v_x, a_x and L. Therefore, $F(t)$ can be specified with:

$$F(t)=F_x(t),\quad (0\le t\le T_x) \tag{18}$$

The expected travel-between $E\,(\mathrm{TB})_{\mathrm{SCAR}}$ time for DCC for two randomly selected locations s_1 and r_1 is equal to the following expression:

$$E(\mathrm{TB})_{\mathrm{SCAR}}=\int_0^{T_x}[1-F(t)]\mathrm{d}t \tag{19}$$

- Expected dual command cycle time for the shuttle carrier

According to travelling of the shuttle carrier in the horizontal direction, the expected dual command cycle time for the SBS/RS is represented with the following expression:

$$E(\mathrm{DCC})_{\mathrm{SCAR}}=4t_{\mathrm{P/S}}+E(\mathrm{SCC})_{\mathrm{SCAR}}+E(\mathrm{TB})_{\mathrm{SCAR}} \tag{20}$$

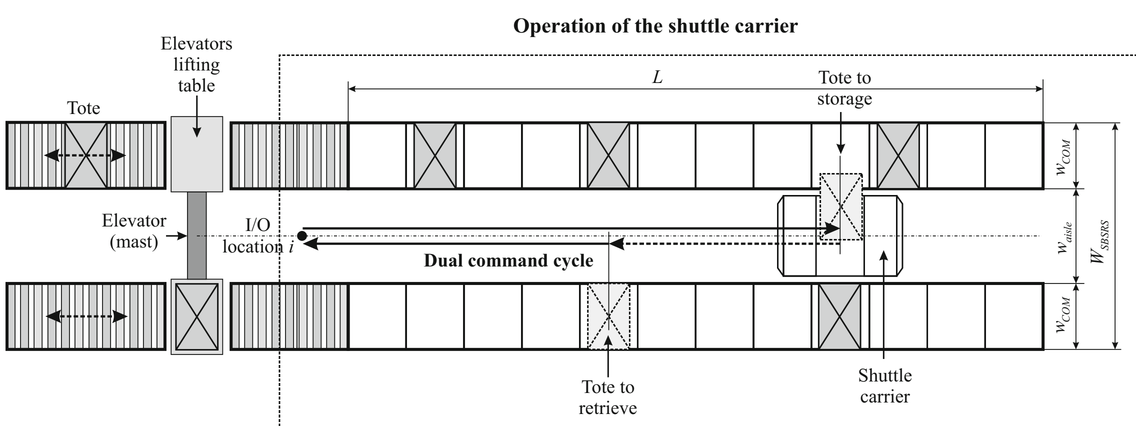

Fig. 7 Dual command cycle of the shuttle carrier operating in the ith tier of the SBS/RS

where $t_{P/S}$ stands for the pick-up and set-down times of the shuttle carrier.

Pick-up and set-down times $t_{P/S}$ of the shuttle carrier in case of DCC relates to pick up the tote from the $I/O_{tier(i)}$ buffer position, set down the tote at the storage location s_1 in the ith tier of the SR, pick up the tote from the retrieval location r_1 in the ith tier of the SR and finally set down the tote at the $I/O_{tier(i)}$ buffer position.

4 Approximation travel time model for SBS/RS

In order to compare the proposed analytical travel time model for SBS/RS, the approximation model for SBS/RS will be short presented. The approximation model for SBS/RS relates to probability theory (lifting of the elevators lifting table in the vertical direction) and application of the FEM guidelines (travelling of the shuttle carrier in the horizontal direction).

4.1 Lifting of the elevators lifting table in the vertical direction

By assuming the distances between tiers to be large enough for the elevators lifting table to achieve its maximum velocity v_{max} and a continuous sequence of loaded and unloaded moves between destination points (tiers), average travel (cycle) time using the probability theory, can be calculated [23].

Figure 8 shows an SBS/RS with m tiers operated by the elevators lifting table. Elevators lifting table can serve m tiers each one with an average height of $h$tier.

Travel time from I/O location to the buffer position at ith tier is calculated by Eq. (21):

$$t_{I/O,tier(i)} = \frac{v_y}{a_y} + \frac{H}{m}\frac{(m+1)}{2v_y} \tag{21}$$

Travel time from the buffer position at ith tier to the buffer position at tier jth is calculated by Eq. (22):

$$t_{tier(i),tier(j)} = \frac{v_y}{a_y} + \frac{H}{m}\frac{(m+1)}{3v_y} \tag{22}$$

- Mean single command cycle time

Mean single command cycle time $T(SCC)_{LIFT}$ is calculated by Eq. (23):

$$\begin{aligned} T(SCC)_{LIFT} &= 2t_{P/S} + 2t_{I/O,tier(i)} \\ T(SCC)_{LIFT} &= 2t_{P/S} + 2\frac{v_y}{a_y} + \frac{h}{v_y}(m+1) \end{aligned} \tag{23}$$

where $t_{P/S}$ stands for the pick-up and set-down times of the elevators lifting table.

- Mean dual command cycle time

Mean dual command cycle time $T(DCC)_{LIFT}$ is calculated by Eq. (24):

$$\begin{aligned} T(DCC)_{LIFT} &= 4t_{P/S} + 2t_{I/O,tier(i)} + t_{tier(i),tier(j)} \\ T(DCC)_{LIFT} &= 4t_{P/S} + 2\frac{v_y}{a_y} + \frac{h}{v_y}(m+1) + 2\frac{v_y}{a_y} + \frac{2}{3}\frac{h}{v_y}(m+1) \end{aligned} \tag{24}$$

where $t_{P/S}$ stands for the pick-up and set-down times of the elevators lifting table.

4.2 Travelling of the shuttle carrier

A shuttle carrier can receive one tote at a time and can operate on a single or dual command cycles (FEM Section IX, 2001; see Fig. 9).

Travel time of the shuttle carrier from the $I/O_{tier(i)}$ to point s_1 is calculated by Eq. (25):

$$\begin{aligned} t_{I/O_{tier(i)},s_1} &= \frac{v_x}{a_x} + \frac{x_1 - x_0}{v_x} \\ t_{I/O_{tier(i)},s_1} &= \frac{v_x}{a_x} + \frac{1L}{5v_x} \end{aligned} \tag{25}$$

Travel time of the shuttle carrier from the point s_1 to the point r_1 is calculated by Eq. (26):

$$\begin{aligned} t_{s_1,r_1} &= \frac{v_x}{a_x} + \frac{x_2 - x_1}{v_x} \\ t_{s_1,r_1} &= \frac{v_x}{a_x} + \frac{7L}{15v_x} \end{aligned} \tag{26}$$

Travel time of the shuttle carrier from the point r_1 to $I/O_{tier(i)}$ is calculated by Eq. (27):

$$\begin{aligned} t_{r_1,I/O_{tier(i)}} &= \frac{v_x}{a_x} + \frac{x_2 - x_0}{v_x} \\ t_{r_1,I/O_{tier(i)}} &= \frac{v_x}{a_x} + \frac{2L}{3v_x} \end{aligned} \tag{27}$$

- Mean single command cycle time

Mean single command cycle time $T\,(SCC)_{SCAR}$ is calculated by Eq. (28):

$$\begin{aligned} T(SCC)_{SCAR} &= 2t_{P/S} + 2t_{I/O_{tier(i)},s_1} \\ T(SCC)_{SCAR} &= 2t_{P/S} + 2\left(\frac{v_x}{a_x} + \frac{1L}{5v_x}\right) \end{aligned} \tag{28}$$

where $t_{P/S}$ stands for the pick-up and set-down times of the shuttle carrier.

- Mean dual command cycle time

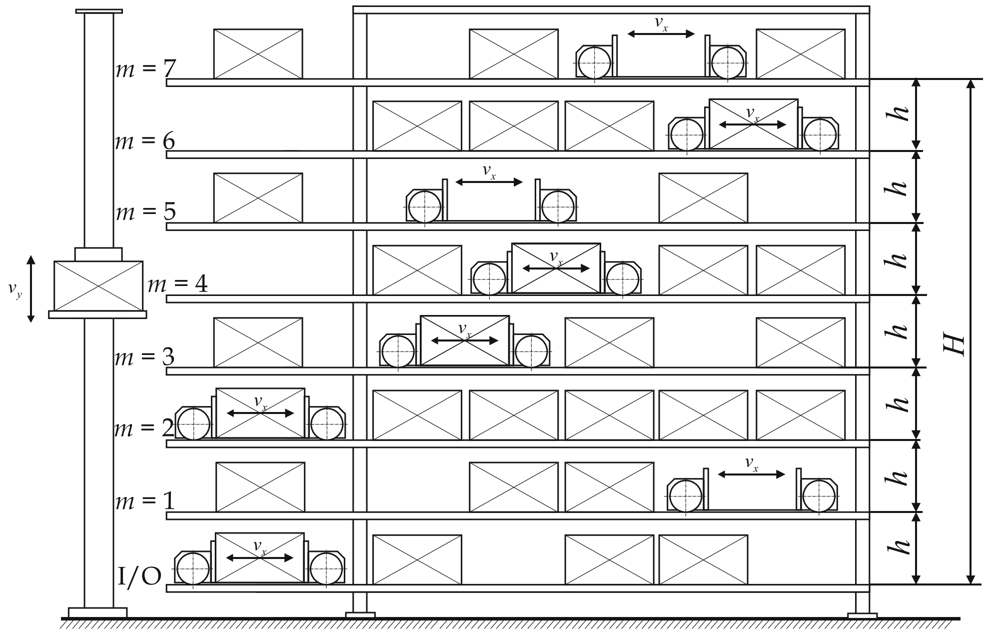

Fig. 8 SBS/RS with m tiers [19]

Mean dual command cycle time $T(\mathrm{DCC})_{\mathrm{SCAR}}$ is calculated by Eq. (29):

$$T(\mathrm{DCC})_{\mathrm{SCAR}} = 4t_{\mathrm{P/S}} + t_{\mathrm{I/O}_{\mathrm{tier}(i)},s_1} + t_{s_1,\,r_1} + t_{r_1,\mathrm{I/O}_{\mathrm{tier}(i)}}$$
$$T(\mathrm{DCC})_{\mathrm{SCAR}} = 4t_{\mathrm{P/S}} + \left(\frac{v_x}{a_x} + \frac{1L}{5v_x}\right) + \left(\frac{v_x}{a_x} + \frac{7L}{15v_x}\right) + \left(\frac{v_x}{a_x} + \frac{2L}{3v_x}\right) \tag{29}$$

where $t_{\mathrm{P/S}}$ stands for the pick-up and set-down times of the shuttle carrier.

5 Simulation model of SBS/RS

To facilitate the performance evaluation and comparison of the proposed analytical travel time model for SBS/RS and the approximation model, discrete event simulation was employed.

Our simulation model begins with the process which marks the whole storage locations in the SBS/RS according to the prescribed storage area. After creating the list of free storage locations, the first tote is entered in the simulation model, which is situated in the input/output (I/O) location of the SBS/RS. Further on, the tote receives a sign, which belongs to the buffer position in the ith tier and the storage location in the SR of the ith tier. The elevators lifting table picks up the tote from the I/O location and moves to the buffer position in the ith tier. After conducting transport to the buffer location in the ith tier, the elevators lifting table set down the tote, which is waiting to be moved by a shuttle carrier. Next, the shuttle carrier picks up the tote from the buffer position in the ith tier and travels to the storage location, where the tote is set down by a shuttle carrier. For the storage operation, the randomised storage policy has been used. Next, the tote that has been stored is put on the waiting list by a computer (computer data base), where it waits for the retrieval operation. For the retrieval process the random request selection rule has been used. After the storage operation in the ith tier, the shuttle carrier travels to the retrieval location in the ith tier of the storage rack. The retrieval location is positioned in the same tier (tier-captive system). Next, the shuttle carrier picks up the tote and moves through the tier to the buffer position in the ith tier, where the tote is picked up by the elevators lifting table.

According to the dwell point strategy, the elevators lifting tables and shuttle carriers always return to the I/O location when they are idle. This strategy is also referred to as the I/O strategy.

The fillgrade factor α of the warehouse (SBS/RS) capacity was set to 0.85, which means that 85 % were closed and 15 % were empty locations.

For every single type of SBS/RS 10,000 runs with different initial conditions were conducted.

The mean cycle time for the transaction is associated with the travel time of the elevators lifting table and the travel time of the shuttle carrier. As a performance measure for the SBS/RS, the mean dual command cycle time has been used.

Movement of the elevators lifting table and travelling of the shuttle carrier in the simulation model is based on the real velocity-time dependence (see Section 3.1).

As mentioned before, dual command cycle requests for the storage and retrieval sequence of the elevators lifting table and for the shuttle carrier are based on the random strategy and I/O dwell-point strategy.

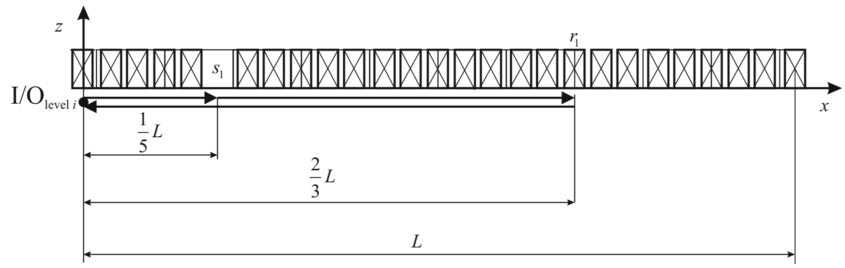

Fig. 9 Modified FEM 9.851 for the shuttle carrier movement in the horizontal direction [19]

A detailed procedure (operation) of the shuttle carrier in SBS/RS is presented with the following algorithm:

Algorithm of the logistics for DCC of the shuttle carrier

1: Selection of one random open storage location in *i*th tier for the storage sequence

2: Selection of one random closed storage locations in *i*th tier for the retrieval sequence

3: Performance of the storage and the retrieval assignment of the shuttle carrier under DCC

$s_i \in (S_i; i=1,\dots,n)$ and $r_i \in (R_i; i=1,\dots,n)$

If $|R_i| \geq 1$, then select $s_i \in S_i$ and $r_i \in R_i$ randomly:

$R_i \leftarrow R_i - \{r_i\}$ $S_i \leftarrow S_i - \{s_i\} + \{r_i\}$

One new open location is created in S_i (r_i), and one open location (s_i) is lost.

According to the algorithm of the logistics for DCC of the shuttle carrier, mean dual command cycle time $T(\text{DCC})_{\text{SCAR}}$ equals: (i) mean travel time from the I/O$_{\text{tier}(i)}$ location to random open (storage) location, (ii) mean travel time from random open (storage) location to random close (retrieval) location and (iii) finally mean travel time from random close (retrieval) location to the I/O$_{\text{tier}(i)}$ location.

Mean dual command cycle time $T(\text{DCC})_{\text{SCAR}}$ is enlarged for all the manipulations related for totes handling (pick-up and set-down times, identification time, etc.)

$$T(\text{DCC})_{\text{SCAR}} = 4t_{\text{P/S}} + t_{\text{I/O}_{\text{tier}(i)},s_1} + t_{s_1,\, r_1} + t_{r_1,\text{I/O}_{\text{tier}(i)}} \tag{30}$$

A detailed procedure (operation) of the elevators lifting table in SBS/RS is presented with the following algorithm:

Algorithm of the logistics for DCC of the elevators lifting table

1: Selection of one random open buffer position at the *i*th tier for the storage sequence

2: Selection of one random closed buffer position at the *j*th tier for the retrieval sequence

3: Performance of the storage and the retrieval assignment of the elevators lifting table under DCC

$s_i \in (S_i; i=1,\dots,n)$ and $r_i \in (R_i; i=1,\dots,n)$

If $|R_i| \geq 1$, then select $s_i \in S_i$ and $r_i \in R_i$ randomly:

$R_i \leftarrow R_i - \{r_i\}$ $S_i \leftarrow S_i - \{s_i\} + \{r_i\}$

One new open location is created in S_i (r_i) and one open location (s_i) is lost.

According to the algorithm of the logistics for DCC of the elevators lifting table, mean dual command cycle time $T(\text{DCC})_{\text{LIFT}}$ equals: (i) mean travel time from I/O location to the random open (storage) buffer position in the *i*th tier, (ii) mean travel time from random open (storage) buffer position in the *i*th tier to the random closed (retrieval) buffer position in the *j*th tier and (iii) finally mean travel time from the random closed (retrieval) buffer position in the *j*th tier to the I/O location.

Mean dual command cycle time T $(\text{DCC})_{\text{LIFT}}$ is enlarged for all the manipulations related for totes handling (pick-up and set-down times, identification time, etc.)

$$T(\text{DCC})_{\text{LIFT}} = 4t_{\text{P/S}} + t_{\text{I/O,tier}(i)} + t_{\text{tier}(i),\text{tier}(j)} + t_{\text{tier}(j),\ \text{I/O}} \tag{31}$$

6 SBS/RS (case study)

In this section, main input data for the analysis are provided and discussed. Stock keeping unit represents a tote (plastic container) filled with items with the dimensions: length l_{tote}=0.6 m, width w_{tote}=0.4 m and height h_{tote}=0.24 m. With regard to the tote, the storage place has the following dimensions: length (depth) of the column l_{COM}=0.6 m, width of the column w_{COM}= 0.5 m and height of the tier h_{COM}=0.35 m. Dimensions of the SBS/RS storage rack (L and H) depends on the number of columns C in the horizontal direction and number of tiers M in the vertical direction, respectively. Pick-up and set-down times for the shuttle carrier were set to $t_{\text{P/S}}$=3.0 s, and for the elevators lifting table were set to $t_{\text{P/S}}$=1.5 s.

As can be seen in Table 1, nine SBS/RS configurations were analysed based on three values of tiers M (M=10, M=15 and M=20) and three values of aisles A (A=3, A=6 and A=9). Total number of storage locations Q is assumed to be approximately 10,000 storage locations [22].

Velocity profiles from vp$_i$ (i=1, …, 5) are selected according to the references of material handling equipment producers and practical experiences of the authors.

6.1 Analyses and evaluation of results

The expected and the mean dual command cycle times (in seconds) for the SBS/RS are given on the basis of the performed analyses. Analyses have been conducted for nine different SBS/RS [22] (see Table 1) with five different velocity profiles (see Table 2) of the elevators lifting table and the shuttle carrier under the condition of the tier-captive SBS/RS. In order to receive the best representative average of the mean dual command cycle time, the simulation results presented in Tables 3 and 4 correspond to 10,000 runs for every single type of SBS/RS.

According to the simulation results presented in Tables 3 and 4, the performance comparison between the proposed analytical model for SBS/RS and the approximation model for SBS/RS has been calculated with the next expression:

$$\lambda = \left[\frac{E(\mathrm{DCC}) \times 100}{T(\mathrm{DCC})}\right] - 100 \tag{32}$$

According to the comparison of the mean dual command cycle time of the elevators lifting table, deviations up to 19 % (vp_1 and vp_2) and deviations up to 23 % (vp_3, vp_4 and vp_5) are noticed in Table 3. This means that the mean dual command cycle time of the elevators lifting table is overestimated, which has a consequence for the throughput capacity of the elevator. Using the approximation model, the throughput capacity of the elevators lifting table will be lower up to 16 % (vp_1 and vp_2) and up to 18 % (vp_3, vp_4 and vp_5). When taking into consideration the shuttle carrier, a changed deviational characteristic of the mean dual command cycle time can be noticed. Percent error between the approximation model and the simulation model is up to 1 % only, which is a good approximation for calculation of the throughput capacity of the shuttle carrier.

According to the results in Table 4, the proposed analytical travel time model for the SBS/RS demonstrates good performances with regard to the results of simulation analysis. In general, according to the difference between the proposed analytical model and the simulation model, small deviations of the mean dual command cycle time of the elevator's lifting table and the shuttle carrier in the range of <1.5 % are noticed. The latter show that the proposed analytical travel time model demonstrates satisfactory deviations of the dual command cycle time and throughput performances for the selected SBS/RS.

7 Conclusion

In this paper, proposed analytical travel time model for SBS/RS is presented. According to approximation travel time model for SBS/RS, where mean uniform velocity is used only, the real operating characteristics for the elevator's lifting table and for the shuttle carrier have been used in the proposed analytical travel time model. In the proposed model we originate from moving of the elevator's lifting table in the vertical direction and travelling of the shuttle carrier in the horizontal direction. Thus, considering both independent movements of the elevator's lifting table and the shuttle carrier, the proposed analytical travel time model for SBS/RS have been developed. The proposed analytical travel time model for SBS/RS deals for the aisle-captive SBS/RS. Various elements of the SBS/RS have been examined, such as the layout of the SR and the velocity profile of the elevator's lifting table and the shuttle carrier in order to investigate the efficiency of the proposed analytical travel time model for SBS/RS in comparison with the simulation model of SBS/RS.

According to the approximation travel time model for SBS/RS, which is based on constant velocity only, the largest deviation of $T(\mathrm{DCC})_{\mathrm{LIFT}}$ occurs in range from 18 to 23 %. Therefore, the evaluated $T(\mathrm{DCC})_{\mathrm{LIFT}}$ is over overestimated, which indicates the difficulty in planning the SBS/RS. On the contrary, the proposed analytical travel time model for SBS/RS shows satisfactory deviations of E(DCC) (max. 1.5 %). Therefore, the proposed analytical travel time model for SBS/RS demonstrates good performances for the SBS/RS and could be a very useful tool for designing SBS/RS. The proposed model could be of considerable help to professionals in practice, when making decisions in the early stages of design project of SBS/RS and when deciding which type of elevators lifting table and shuttle carrier will be the most promising.

Appendices

Appendix A: Verification of expressions 2 and 4

- Verification of expression 2 where the peak velocity v_p is less than $v_{\max}$.

The velocity in dependence of time $v(t)$ equals the following expression [18]:

$$v(t) = \begin{cases} at, & t \in (0, t_p) \\ -a(t-T), & t \in (t_p, T) \end{cases} \tag{33}$$

Distances in dependence of time $d_1(T)$ and $d_2(T)$ according to conditions 1 and 2 equal the following expressions (see Fig. 3):

- Condition 1: $0 \le t \le t_p$

$$\begin{aligned} a(t) &= a \\ v(t) &= at \\ d_1(t) = \int_0^{t_p} v(t)\mathrm{d}t &= \int_0^{t_p} at\mathrm{d}t = a\frac{t_p^2}{2} \end{aligned} \tag{34}$$

Table 1 SBS/RS configurations

SBS/RS configuration (RC)	Number of tiers (M)	Number of aisles (A)	Number of columns (C)	Length of the storage rack (L)	Height of the storage rack (H)	Warehouse volume (Q)
1	10	3	167	83.50	3.50	10,020
2	10	6	84	42.00	3.50	10,080
3	10	9	56	28.00	3.50	10,080
4	15	3	112	56.00	5.25	10,080
5	15	6	56	28.00	5.25	10,080
6	15	9	38	19.00	5.25	10,260
7	20	3	84	42.00	7.00	10,080
8	20	6	42	21.00	7.00	10,080
9	20	9	28	14.00	7.00	10,080

- Condition 2: $t_p \leq t \leq T$

$$\begin{gathered} a(t) = a \\ v(t) = -a(t-T) \\ d_2(t) = \int_{t_p}^{T} v(t)\mathrm{d}t = \int_{t_p}^{T} -a(t-T)\mathrm{d}t = \frac{1}{2}a\left(T-t_p\right)^2 \end{gathered} \tag{35}$$

The distance in dependence of time $d(T)$ equals the following expression:

$$d(T) = d_1(t) + d_2(t) = a\frac{t_p^2}{2} + \frac{1}{2}a\left(T-t_p\right)^2 \tag{36}$$

Considering the condition $t_p=T/2$, the distance in dependence of time $d(T)$ equals the next expression:

$$d(T) = \frac{aT^2}{4} \tag{37}$$

- Verification of expression 4 where the peak velocity v_p is equal to $v_{\max}$.

Table 2 Velocity scenarios of the shuttle carrier and the elevators lifting table

Velocity profile	Shuttle carrier travelling in the horizontal direction			Lifting table movement in the vertical direction		
	v_x (m/s)	a_x^+ (m/s^2)	a_x^- (m/s^2)	v_y (m/s)	a_y^+ (m/s^2)	a_y^- (m/s^2)
vp$_1$	1.5	1.5	1.5	1.5	1.5	1.5
vp$_2$	2.0	1.5	1.5	1.5	1.5	1.5
vp$_3$	3.0	2.0	2.0	2.0	1.5	1.5
vp$_4$	3.0	3.0	3.0	2.0	1.5	1.5
vp$_5$	4.0	3.0	3.0	2.0	1.5	1.5

The velocity in dependence of time $v(t)$ equals the following expression [18]:

$$v(t) = \begin{cases} at, & t\in\left(0, t_p\right) \\ v_{\max}, & t\in\left(t_p, T-t_p\right) \\ -a(t-T), & t\in\left(T-t_p, T\right) \end{cases} \tag{38}$$

Distances in dependence of time $d_1(T)$, $d_2(T)$ and $d_3(T)$ according to conditions 1, 2 and 3 equal the following expressions (see Fig. 3):

- Condition 1: $0 \leq t \leq t_p$

$$\begin{gathered} a(t) = a \\ v(t) = at \\ d_1(t) = \int_0^{t_p} v(t)\mathrm{d}t = \int_0^{t_p} at\mathrm{d}t = \frac{at_p^2}{2} \end{gathered} \tag{39}$$

- Condition 2: $t_p \leq t \leq T-t_p$

$$\begin{gathered} a(t) = 0 \\ v(t) = v_{\max} \\ d_2(t) = \int_{t_p}^{T-t_p} v(t)\mathrm{d}t = \int_{t_p}^{T-t_p} v_{\max}\mathrm{d}t = v_{\max}\left(T-2t_p\right) \end{gathered} \tag{40}$$

- Condition 3: $T-t_p \leq t \leq T$

$$\begin{gathered} a(t) = a \\ v(t) = -a\left(T-t_p, T\right) \\ d_3(t) = \int_{T-t_p}^{T} v(t)\mathrm{d}t = \int_{T-t_p}^{T} -a(T-t)\mathrm{d}t = \frac{at_p^2}{2} \end{gathered} \tag{41}$$

The distance in dependence of time $d(T)$ equals the following expression [18]:

Table 3 Performance comparison of approximation model of SBS/RS according to the simulation model of SBS/RS

vp_i	RC*i*	Elevator			Shuttle carrier		
		Approximation $T(DCC)_{LIFT}$	Percent error	Simulation $T(DCC)_{LIFT}$	Approximation $T(DCC)_{SCAR}$	Percent error	Simulation $T(DCC)_{SCAR}$
vp_1	1	13.89[a]	(18.72)[b]	11.70[a]	89.22[a]	(−0.31)[b]	89.5[a]
	2	13.89	(18.72)	11.70	52.33	(−0.13)	52.4
	3	13.89	(18.72)	11.70	39.89	(−0.03)	39.9
	4	15.83	(19.02)	13.30	64.78	(−0.18)	64.9
	5	15.83	(19.02)	13.30	39.89	(−0.03)	39.9
	6	15.83	(19.02)	13.30	31.89	(−0.34)	32.0
	7	17.78	(18.53)	15.00	52.33	(−0.13)	52.4
	8	17.78	(18.53)	15.00	33.67	(−0.68)	33.9
	9	17.78	(18.53)	15.00	27.44	(−0.22)	27.5
vp_2	1	13.89	(18.72)	11.70	71.67	(−0.32)	71.9
	2	13.89	(18.72)	11.70	44.00	(−0.23)	44.1
	3	13.89	(18.72)	11.70	34.67	(0.20)	34.6
	4	15.83	(19.02)	13.30	53.33	(−0.13)	53.4
	5	15.83	(19.02)	13.30	34.67	(0.20)	34.6
	6	15.83	(19.02)	13.30	28.67	(−0.10)	28.7
	7	17.78	(18.53)	15.00	44.00	(−0.23)	44.1
	8	17.78	(18.53)	15.00	30.00	(−0.33)	30.1
	9	17.78	(18.53)	15.00	25.33	(0.12)	25.3
vp_3	1	14.25	(22.84)	11.60	53.61	(−0.17)	53.7
	2	14.25	(22.84)	11.60	35.17	(0.20)	35.1
	3	14.25	(22.84)	11.60	28.94	(0.49)	28.8
	4	15.71	(21.78)	12.90	41.39	(−0.02)	41.4
	5	15.71	(21.78)	12.90	28.94	(0.49)	28.8
	6	15.71	(21.78)	12.90	24.94	(0.56)	24.8
	7	17.17	(20.07)	14.30	35.17	(0.20)	35.1
	8	17.17	(20.07)	14.30	25.83	(0.12)	25.8
	9	17.17	(20.07)	14.30	22.72	(0.98)	22.5
vp_4	1	14.25	(22.84)	11.60	52.11	(−0.17)	52.2
	2	14.25	(22.84)	11.60	33.67	(−0.09)	33.7
	3	14.25	(22.84)	11.60	27.44	(0.15)	27.4
	4	15.71	(21.78)	12.90	39.89	(−0.03)	39.9
	5	15.71	(21.78)	12.90	27.44	(0.15)	27.4
	6	15.71	(21.78)	12.90	23.44	(0.17)	23.4
	7	17.17	(20.07)	14.30	33.67[a]	(−0.09)	33.7
	8	17.17	(20.07)	14.30	24.33	(−0.29)	24.4
	9	17.17	(20.07)	14.30	21.22	(0.09)	21.2
vp_5	1	14.25	(22.84)	11.60	43.83	(−0.16)	43.9
	2	14.25	(22.84)	11.60	30.00	(0.00)	30.0
	3	14.25	(22.84)	11.60	25.33	(0.52)	25.2
	4	15.71	(21.78)	12.90	34.67	(−0.09)	34.7
	5	15.71	(21.78)	12.90	25.33	(0.52)	25.2
	6	15.71	(21.78)	12.90	22.33	(0.59)	22.2
	7	17.17	(20.07)	14.30	30.00	(0.00)	30.0
	8	17.17	(20.07)	14.30	23.00	(0.44)	22.9
	9	17.17	(20.07)	14.30	20.67	(1.32)	20.4

[a] The cycle time associated with load handling measured in seconds

[b] Percent error between the approximation model and the discrete model

$$d(T) = d_1(t) + d_2(t) + d_3(t) = \frac{at_p^2}{2} + v_{\max}\left(T-2t_p\right) + \frac{at_p^2}{2} \quad (42)$$

Considering the condition $t_p = v_{max}/a$, the distance in dependence of time $d(T)$ equals the next expression [18]:

$$d(T) = v_{\max}T - \frac{v_{\max}^2}{a} \quad (43)$$

Appendix B: Verification of expressions 5 and 9

- Verification of expression 5

Let the storage (or retrieval) point of the elevators lifting table be represented by $P(y)$ where $0 \le y \le H$. Travel time t_{ky} from the I/O location is the variable share of the vertical travel time. Depending of the regions for the travel type I and II (see Section 3.1), the vertical travel time T_{ky} equals the following expression:

$$\begin{aligned} T_{ky} &= \tau_y\left[y_{(k)}\right] \\ t_{ky} &= \tau_y\left[y_{(k)}\right] \end{aligned} \quad (44)$$

where t_{ky} is the variable share of the vertical travel time, $y_{(k)}$ coordinates of location k and $\tau_y[y_{(k)}]$ is the travel time function of the elevators lifting table.

Let $F_{T_{ky}}(t)$ denotes the probability that the travel time to $y_{(k)}$ is less than or equal to t.

$$F_{T_{ky}}(t) = \Pr\left(T_{ky} \le t\right) = \Pr\left[\tau_y\left(y_{(k)}\right) \le t\right] \quad (45)$$

Regarding the condition that the function $\tau_y[y_{(k)}]$ is monotony increased, it can be copied. Therefore, $F_{T_{ky}}(t)$ equals the following expression:

$$F_{T_{ky}}(t) = \Pr\left[\left(y_{(k)}\right) \le \tau_y^{-1}(t)\right] \quad (46)$$

For the randomised storage assignment rule, it can be assumed that the location y is uniformly distributed. Therefore, $F_{T_{ky}}(t)$ equals the following expression:

$$f_z(z) = \begin{cases} 1 & 0 \le z \le 1 \\ 0 & \text{otherwise} \end{cases} \quad (47)$$

$$F_{T_{ky}}(t) = \int_{-\infty}^{\tau_y^{-1}(t)} f_z(z)\mathrm{d}z = \int_{-\infty}^{\tau_y^{-1}(t)} 1\mathrm{d}z = \tau_y^{-1}(t) \quad (48)$$

The inverse function $\tau_y^{-1}[y_{(k)}]$ is derived from the relationship of the travel time as a function of displacement (distance travelled) and thus equals the relationship of displacement (distance travelled) as a function of travel time.

$$\tau_k^{-1}(t) = s(t) \quad (49)$$

It can be shown that:

$$F_{T_{ky}}(t) = s(t) \quad (50)$$

Normalising the height H on the unit (1), the cumulative distribution function $F_{T_{ky}}(t)$ represents the following relationship.

The cumulative distribution functions are distinguished according to the following condition: (i) elevators lifting table moving for type I (see Section 3.1) and (ii) elevators lifting table moving for type II (see Section 3.1).

Type 1:

$$\begin{aligned} &0 \le t \le \frac{2v_y}{a_y} \\ &F_{\mathrm{I}y}(t) = \frac{t^2 a_y}{4H} \end{aligned} \quad (51)$$

Type II:

$$\begin{aligned} &\frac{2v_y}{a_y} \le t \le \frac{v_y}{a_y} + \frac{H}{v_y} \\ &F_{\mathrm{II}y}(t) = \frac{v_y t}{H} - \frac{{v_y}^2}{a_y H} \end{aligned} \quad (52)$$

- Verification of expression 9

Let the storage (or retrieval) location of the shuttle carrier be represented by $P(x)$ where $0 \le x \le L$. Travel time t_{kx} from the I/O$_{\text{tier}(i)}$ is the variable share of the horizontal travel time. Depending of the regions for the travel type I and II (see Section 3.1), the horizontal travel time T_{kx} equals the following expression:

$$\begin{aligned} T_{kx} &= \tau_x\left[x_{(k)}\right] \\ t_{kx} &= \tau_x\left[x_{(k)}\right] \end{aligned} \quad (53)$$

where t_{kx} is the variable share of the horizontal travel time, $x_{(k)}$ coordinate of location k and $\tau_x(y_{(k)})$ is the travel time function of the shuttle carrier.

Let $F(t)$ denotes the probability that the travel time to $x_{(k)}$ is less than or equal to t.

$$F_{T_{kx}}(t) = \Pr(T_{kx} \le t) = \Pr\left[\tau_x\left[x_{(k)}\right] \le t\right] \quad (54)$$

Regarding the condition that the function $\tau_x[y_{(k)}]$ is monotony increased, it can be copied. Therefore:

Table 4 Performance comparison of proposed analytical travel time model of SBS/RS according to the simulation model of SBS/RS

vp_i	RC*i*	Elevator			Shuttle carrier		
		Analytical E $(DCC)_{LIFT}$	Percent error	Simulation T $(DCC)_{LIFT}$	Analytical E $(DCC)_{SCAR}$	Percent error	Simulation T $(DCC)_{SCAR}$
vp_1	1	11.83[a]	(1.11)[b]	11.70[a]	89.21[a]	(−0.32)[b]	89.5[a]
	2	11.83	(1.11)	11.70	52.30	(−0.19)	52.4
	3	11.83	(1.11)	11.70	39.85	(−0.13)	39.9
	4	13.48	(1.35)	13.30	64.67	(−0.35)	64.9
	5	13.48	(1.35)	13.30	39.85	(−0.13)	39.9
	6	13.48	(1.35)	13.30	31.84	(−0.50)	32.0
	7	15.09	(0.60)	15.00	52.30	(−0.19)	52.4
	8	15.09	(0.60)	15.00	33.62	(−0.83)	33.9
	9	15.09	(0.60)	15.00	27.38	(−0.44)	27.5
vp_2	1	11.83	(1.11)	11.70	71.64	(−0.36)	71.9
	2	11.83	(1.11)	11.70	43.95	(−0.34)	44.1
	3	11.83	(1.11)	11.70	34.58	(−0.06)	34.6
	4	13.48	(1.35)	13.30	53.29	(−0.21)	53.4
	5	13.48	(1.35)	13.30	34.58	(−0.06)	34.6
	6	13.48	(1.35)	13.30	28.54	(−0.56)	28.7
	7	15.09	(0.60)	15.00	43.94	(−0.36)	44.1
	8	15.09	(0.60)	15.00	29.88	(−0.73)	30.1
	9	15.09	(0.60)	15.00	25.16	(−0.55)	25.3
vp_3	1	11.70	(0.86)	11.60	53.55	(−0.28)	53.7
	2	11.70	(0.86)	11.60	35.06	(−0.11)	35.1
	3	11.70	(0.86)	11.60	28.78	(−0.07)	28.8
	4	13.07	(1.32)	12.90	41.30	(−0.24)	41.4
	5	13.07	(1.32)	12.90	28.78	(−0.07)	28.8
	6	13.07	(1.32)	12.90	24.71	(−0.36)	24.8
	7	14.34	(0.28)	14.30	35.06	(−0.11)	35.1
	8	14.34	(0.28)	14.30	25.63	(−0.66)	25.8
	9	14.34	(0.28)	14.30	22.41	(−0.40)	22.5
vp_4	1	11.70	(0.86)	11.60	52.08	(−0.23)	52.2
	2	11.70	(0.86)	11.60	33.62	(−0.24)	33.7
	3	11.70	(0.86)	11.60	27.37	(−0.11)	27.4
	4	13.07	(1.32)	12.90	39.86	(−0.10)	39.9
	5	13.07	(1.32)	12.90	27.38	(−0.07)	27.4
	6	13.07	(1.32)	12.90	23.34	(−0.26)	23.4
	7	14.34	(0.28)	14.30	33.62	(−0.24)	33.7
	8	14.34	(0.28)	14.30	24.24	(−0.66)	24.4
	9	14.34	(0.28)	14.30	21.08	(−0.57)	21.2
vp_5	1	11.70	(0.86)	11.60	43.78	(−0.27)	43.9
	2	11.70	(0.86)	11.60	29.88	(−0.40)	30.0
	3	11.70	(0.86)	11.60	25.17	(−0.12)	25.2
	4	13.07	(1.32)	12.90	34.58	(−0.35)	34.7
	5	13.07	(1.32)	12.90	25.16	(−0.16)	25.2
	6	13.07	(1.32)	12.90	22.09	(−0.50)	22.2
	7	14.34	(0.28)	14.30	29.88	(−0.40)	30.0
	8	14.34	(0.28)	14.30	22.78	(−0.52)	22.9
	9	14.34	(0.28)	14.30	20.34	(−0.29)	20.4

[a] The cycle time associated with load handling measured in seconds

[b] Percent error between the proposed model and the discrete model

$$F_{T_{kx}}(t) = \Pr\left[\left(x_{(k)}\right) \leq \tau_x^{-1}(t)\right] \tag{55}$$

For the randomised storage assignment rule, it can be assumed that the location x is uniformly distributed. Therefore:

$$f_z(z) = \begin{cases} 1 & 0 \leq z \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{56}$$

$$F_{T_{kx}}(t) = \int_{-\infty}^{\tau_x^{-1}(t)} f_z(z)\mathrm{d}z = \int_{-\infty}^{\tau_x^{-1}(t)} 1\mathrm{d}z = \tau_x^{-1}(t) \tag{57}$$

The inverse function $\tau_x^{-1}[x_{(k)}]$ is derived from the relationship of the travel time as a function of displacement (distance travelled) and thus equals the relationship of displacement (distance travelled) as a function of travel time.

$$\tau_k^{-1}(t) = s(t) \tag{58}$$

It can be shown that:

$$F_{T_k}(t) = s(t) \tag{59}$$

Normalising the length L on the unit (1), the cumulative distribution function $F_{T_{kx}}(t)$ represents the following relationship.

The cumulative distribution functions are distinguished according to the following condition: (i) shuttle carrier travelling for type I (see Section 3.1) and (ii) shuttle carrier travelling for type II (see Section 3.1).

Type I:

$$\begin{aligned} &0 \leq t \leq \frac{2v_x}{a_x} \\ &F_{\mathrm{I}x}(t) = \frac{t^2 a_x}{4L} \end{aligned} \tag{60}$$

Type II:

$$\begin{aligned} &\frac{2v_x}{a_x} \leq t \leq \frac{v_x}{a_x} + \frac{L}{v_x} \\ &F_{\mathrm{II}x}(t) = \frac{v_x t}{L} - \frac{{v_x}^2}{a_x L} \end{aligned} \tag{61}$$

Appendix C: Verification of expressions 13 and 17

- Verification of expression 13

By definition, each DCC involves two random locations, one representing the storage location $P_1(y_1)$ and the other representing the retrieval location $P_2(y_2)$. Let t_{By} be the vertical interleave time required to travel between the storage and the retrieval locations.

The cumulative distribution function $F(t)$ of the interleave time is equal to the following expression:

$$F(t) = \Pr\left(t_{By} \leq t\right) \tag{62}$$

Letting $r_y = \left|y_1 - y_2\right|$ follow:

$$\Pr\left(t_{By} \leq t\right) = \Pr\left(t_{By} \leq t,\ 1 \leq r_y \leq H\right) \tag{63}$$

From expressions 2 and 4 (see Section 3.1), the relation between t_{By} and r_y can be found

$$t_{By} = \left\{ \begin{array}{ll} \sqrt{\dfrac{4r_y}{a}} & \text{for } 0 \leq r_y \leq h \\ \dfrac{r_y}{v_y} + \dfrac{v_y}{a} & \text{for } \mathrm{h} \leq r_y \leq H \end{array} \right\} \tag{64}$$

Utilising equation follows:

$$\Pr\left(t_{By} \leq t\right) = \begin{cases} \Pr\left(r_y \leq \dfrac{at^2}{4}\right. & 0 \leq t \leq t_y \\ \Pr\left(r_y \leq v_y t - \dfrac{v_y^2}{a}\right. & t_y \leq t \leq T_y \end{cases} \tag{65}$$

According to the distribution of a range of a sample [1] and randomised storage policy, the probability density function is achieved:

$$h\left(r_y\right) = \frac{2}{H^2}\left(H - r_y\right) \quad \text{for } 0 \leq r_y \leq H \tag{66}$$

The cumulative distribution functions are distinguished according to the following condition: (i) elevators lifting table moving for type I (see Section 3.1) and (ii) elevators lifting table moving for type II (see Section 3.1).

Type I:

$$\begin{aligned} &0 \leq t \leq \frac{2v_y}{a_y} \\ &\mathrm{P}\left(t_{By} \leq t\right) = \int_0^{\frac{a_y t^2}{4}} \frac{2}{H^2}\left(H - r_y\right)\mathrm{d}r_y \end{aligned} \tag{67}$$

$$F_{\mathrm{I}y}(t) = \frac{a_y}{2H}t^2 - \frac{a_y^2}{16H^2}t^4 \tag{68}$$

Type II:

$$\frac{2v_y}{a_y} \le t \le \frac{v_y}{a_y} + \frac{H}{v_y}$$

$$\mathrm{P}(t_{By} \le t) = \int_0^{v_y t - \frac{v_y^2}{a_y}} \frac{2}{H^2}(H - r_y)\mathrm{d}r_y \tag{69}$$

$$F_{\mathrm{II}y}(t) = -\frac{v_y^2}{H^2}t^2 + \left(\frac{2v_y^3}{a_y H^2} + \frac{2v_y}{H}\right)t - \frac{2v_y^2}{a_y H} - \frac{v_y^4}{a_y^2 H^2} \tag{70}$$

- Verification of expression 17

By definition, each DCC involves two random locations, one representing the storage location $P_1(x_1)$ and the other representing the retrieval location $P_2(x_2)$. Let t_{Bx} be the horizontal interleave time required to travel between the storage and the retrieval locations.

The cumulative distribution function $F(t)$ of the interleave time is equal to the following expression:

$$F(t) = \Pr(t_{Bx} \le t) \tag{71}$$

Letting, $r_x = |x_1 - x_2|$ follow:

$$\Pr(t_{Bx} \le t) = \Pr(t_{Bx} \le t,\ 1 \le r_x \le L) \tag{72}$$

From expressions 2 and 4 (see Section 3.1), the relation between t_{Bx} and r_x can be found

$$t_{Bx} = \begin{Bmatrix} \sqrt{\frac{4r_x}{a_x}} & \text{for } 0 \le r_x \le l \\ \frac{r_x}{v_x} + \frac{v_x}{a_x} & \text{for } 1 \le r_x \le L \end{Bmatrix} \tag{73}$$

Utilising equation follows:

$$\Pr(t_{Bx} \le t) = \begin{cases} \Pr\left(r_x \le \frac{a_x t^2}{4}\right. & 0 \le t \le t_x \\ \Pr\left(r_x \le v_x t - \frac{v_x^2}{a_x}\right. & t_x \le t \le T_x \end{cases} \tag{74}$$

According to the distribution of a range of a sample [1] and randomised storage policy, the probability density function is achieved:

$$h(r_x) = \frac{2}{L^2}(L - r_x) \quad \text{for } 0 \le r_x \le L \tag{75}$$

The cumulative distribution functions are distinguished according to the following condition: (i) shuttle carrier travelling for type I (see Section 3.1) and (ii) shuttle carrier travelling for type II (see Section 3.1).

Type I:

$$0 \le t \le \frac{2v_x}{a_x}$$

$$\mathrm{P}(t_{Bx} \le t) = \int_0^{\frac{a_x t^2}{4}} \frac{2}{L^2}(L - r_x)\mathrm{d}r_x \tag{76}$$

$$F_{\mathrm{I}x}(t) = \frac{a_x}{2L}t^2 - \frac{a_x^2}{16L^2}t^4 \tag{77}$$

Type II:

$$\mathrm{P}(t_{Bx} \le t) = \int_0^{v_x t - \frac{v_x^2}{a_x}} \frac{2}{L^2}(L - r_x)\mathrm{d}r_x \tag{78}$$

$$F_{\mathrm{II}x}(t) = -\frac{v_x^2}{L^2}t^2 + \left(\frac{2v_x^3}{a_x L^2} + \frac{2v_x}{L}\right)t - \frac{2v_x^2}{a_x L} - \frac{v_x^4}{a_x^2 L^2} \tag{79}$$

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